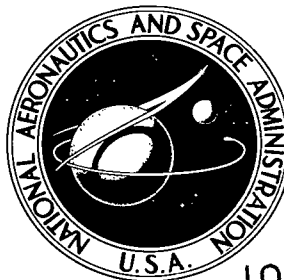


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# OPTICALLY INDUCED FREE-CARRIER LIGHT MODULATOR

*by C. Gruber and W. E. Richards*

*Goddard Space Flight Center  
Greenbelt, Md.*

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • JUNE 1968





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## ABSTRACT

A new technique for modulation of high-power laser energy is described and theoretically analyzed.

It is shown that this technique:

1. Requires no electrical contacts to the modulating medium,
2. Requires low drive voltages,
3. Requires a fraction (approximately  $10^{-3}$ ) of the drive power required by Pockel's effect modulators for performance with high modulation index,
4. Has a capability for beam deflection,
5. Is capable of AM, FM, or PM modulation,
6. Is applicable to optical energy from 6000 Å to millimeter waves.

Computer solutions to the derived wave equations for a single 10.6-micron form of the modulator are included.

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# OPTICALLY INDUCED FREE-CARRIER LIGHT MODULATOR

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## INTRODUCTION

Laser modulators for optical communications systems have recently been the subject of great effort especially in visible or near-infrared regions. The availability of optical sources in these regions, where excellent atmospheric transmission "windows" exist, has prompted attempts to extend the previously developed modulation techniques to longer wavelengths. This has been difficult, because materials having the required optical properties do not generally transmit infrared radiation. Existing devices have other disadvantages such as excessive drive power requirements for high-frequency/wide-bandwidth operation, multicomponent optical configurations, inability to modulate at high optical power levels, and (with one or two exceptions) low modulation index.

The present paper describes an optical modulation technique that should be able to overcome many of these difficulties and, in addition, provide complete electrical isolation of the modulation element (electrical connection to the device sometimes being impossible or undesirable). In addition the technique can be used over a broad range of infrared wavelengths determined solely by the optical transmission characteristics of the modulator material. In brief, this technique uses optically generated free electrons (and/or holes) in a bulk semiconductor in order to change its transmission characteristics. Apart from its construction in suitable semiconducting material, the modulator requires an easily modulated optical pump signal.

## BACKGROUND

### General Review

Before going into the physics of this technique, this paper presents a brief review of the optical modulators now available and their generic disadvantages. Foremost among optical modulators designed for visible and near-infrared operation are the electro-optic devices. These give (relatively to other modulators) linear amplitude and phase modulation with large modulation indices. Wide-bandwidth operation does not demand too much modulation drive power in the visible spectrum, but it demands prohibitive drive power in the infrared region (940 w dc for 60-percent AM over 10-MHz bandwidth), because the power is proportional to the square of the wavelength. Furthermore,

electro-optic materials that transmit radiation at wavelengths over 6 microns are not readily available. (For example, gallium arsenide is at present the only material in general use at 10 microns.) The cost of large single crystals is formidable; also, the long crystal lengths required make the system very difficult to align.

Magneto-optic modulation offers most of the advantages without the required high drive-power of the electro-optic modulator, but material-procurement problems are severe. This modulator, a magnetic analog of the electro-optic device, is still in the developmental stage and has only been used for amplitude modulation.

Band-edge, semiconductor junction modulators, and surface state-free carrier trapping modulators have been constructed that eliminate the high drive-power requirements but suffer from severe disadvantages such as low available modulation indices. Once again, use of these modulators is severely limited in the infrared. Modulators using various scattering or acousto-optic mechanisms are being developed. Devices using interference effects have been developed but are not in general use.

## Basic Operation

Virtually all existing optical modulators are of the variable transmission type, producing amplitude modulation or, less frequently, phase modulation. In this class fall the electro- and magneto-optic modulators with analyzer-polarizer adjuncts, the "junction waveguide" modulator, the bulk-effect devices, the interference-effect modulators, and (broadly interpreted) the devices using optical scattering. The electro- and magneto-optic, junction-waveguide, and interference modulators (as well as the device to be described herein) can also provide phase modulation under certain conditions. Some of the phase modulators can provide narrow-band frequency modulation if used inside the laser cavity. In almost all cases, the modulators answer to this description: The input beam to be modulated, of amplitude intensity  $I_0$ , emerges from the modulator as a beam of intensity  $I = T^2(M)I_0$ , with  $|T(M)| \leq 1$ .  $T(M)$  is the transmission coefficient of the modulator, which depends on the magnitude  $M$  of externally applied modulation signal. For phase modulation, the phase variation of  $T(M)$  is used rather than the amplitude.

## Bulk-Effect Modulators

The transmission coefficient,  $T$ , of bulk materials such as semiconductors depends primarily on two factors: the chemical structure (which determines the energy band structure) and the quasi-free particle concentration (electrons and/or holes). When  $T$  for a particular semiconducting material is plotted as a function of wavelength, the attenuation due to each factor appears in well separated regions of the curve. The wavelengths corresponding to free-carrier absorption generally fall between the intermediate and the far infrared. As the free-carrier density in the material is varied, the absorption coefficient at a fixed wavelength changes in direct proportion. The relative dielectric constant and the index of refraction of the medium are also affected by the presence of free carriers. As the carrier density (and the attenuation coefficient) is increased to a value near

"plasma resonance," the index of refraction is rapidly reduced. This is the effect to be used in the optical modulator described herein. At the other end of the spectrum, band-edge modulators have been devised that use various methods to shift the energy level of the main-band gap of the material and hence vary the absorption coefficient. This technique has not proved very useful, because of difficulties in device fabrication and a resultant low modulation index.

Another method also uses free carrier absorption but reduces the carrier density by trapping electrons in surface states. An applied electric field induces trapping and reduces carrier density in direct proportion. This technique has provided low modulation indices and requires a multiple-reflection geometry.

## Optically Induced Free-Carrier Modulator

This modulator system consists of two components: a source of band-gap resonance radiation, or pump, which can be modulated electrically (such as a semiconductor diode laser), and a thin slab of material that absorbs the resonance radiation but initially transmits the infrared signal source to be modulated (the 10.6-micron wavelength of the  $\text{CO}_2$  laser is of special interest). In the absence of the pump signal, the material should be reasonably transparent to the long infrared wavelength.

The pump source illuminates a small area of material, generating free carriers through electron-hole pair production. The presence of a distribution of free electrons in turn changes the infrared transmission coefficient of the material. Superimposing the pump source and signal beams on the surface of the material and modulating the pump power level causes the signal intensity passing through the material to be synchronously modulated. The basic modulator configuration is shown in Figure 1. The modulation process and further ramifications of the "cross-modulation" effect are presented in detail in the following sections.

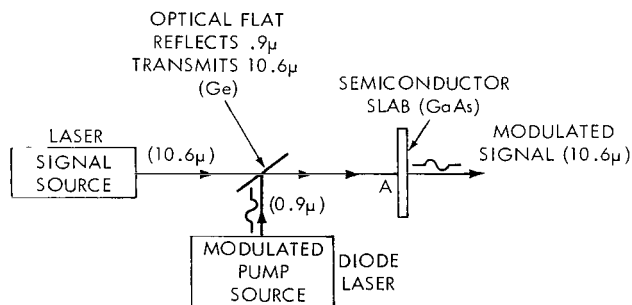


Figure 1—Basic modulator configuration.

## THEORY

### Carrier Generation

Consider a slab of semiconducting material of area,  $A$ , uniformly illuminated by  $P_0$  watts of normally incident pump radiation, greater than or equal to the slab's band-gap energy. The radiation to be modulated at radian frequency  $\omega$  is also assumed normally incident on the same area  $A$  as shown in Figure 1. Since the pump photon energy  $h\nu$  at least equals the band-gap energy  $E_g$ , the

pump power is strongly absorbed in the material according to the relation

$$P = P_0 e^{-2\alpha z} , \quad (1)$$

where  $P_0$  is the incident power level,  $P$  is the power level at a distance  $z$  from the front surface of the slab, and  $\alpha$  is the absorption coefficient. Considering GaAs, for instance,  $\alpha = 2 \times 10^4 (\text{h}\nu)^{1/2}$   $\text{eV/cm}$  near room temperature; this value may be expected to decrease somewhat as the temperature increases.

The incident radiation creates electron-hole pairs with a quantum efficiency  $\eta$  pairs/photon. As a result,  $N_g$  pairs/ $\text{m}^3/\text{sec}$  are produced in an infinitesimal volume of material lying between the points  $z_0$  and  $z_0 + \Delta z$  with  $P_0/\text{h}\nu$  photons/sec incident. The pair-production rate is obtained from Equation 1 as,

$$\begin{aligned} \lim_{\Delta z \rightarrow 0} \left[ \frac{\eta}{\text{h}\nu} \left( P_0 e^{-2\alpha z_0} - P_0 e^{-2\alpha(z_0 + \Delta z)} \right) \right] &= N_g \Delta z A , \\ \lim_{\Delta z \rightarrow 0} \left[ \frac{\eta P_0}{\text{h}\nu} e^{-2\alpha z_0} (2\alpha \Delta z) \right] &= N_g \Delta z A ; \end{aligned}$$

Furthermore, as  $\Delta z \rightarrow 0$ , these equations yield

$$N_g(z) = \frac{\eta P_0}{\text{h}\nu A} (2\alpha) e^{-2\alpha z} \frac{\text{pairs}}{\text{m}^3 \text{ sec}} , \quad (2)$$

where the photon energy  $\text{h}\nu$  is in joules.

The generation process is strongly non-homogeneous, creating large density gradients in the  $z$  direction with attendant particle diffusion. In addition to the diffusion current away from the front surface of the slab, electron-hole recombination provides a loss mechanism for the generated free carriers. Conduction currents can be produced by applying external electric or magnetic fields to the material; and they all change the particle density distribution. The temporal (steady-state to be assumed later) and spatial distribution of either electrons or holes may be determined by solving the continuity equation, using Equation 2 as the source term. The continuity equation (Reference 1), with  $N_g$  an arbitrary function of time, is

$$\frac{d(N - N_0)}{dt} = \frac{\nabla \cdot \vec{J}}{q} - \frac{(N - N_0)}{\tau} + N_g , \quad (3)$$

where  $N_0$  is the carrier density in the absence of  $P_0$ .

The current density,  $\vec{J}$ , in general consists of two terms due to diffusion and conduction, respectively:  $\vec{J}_D = q D \nabla(N - N_0)$  and  $\vec{J}_c = N q \mu \vec{E}$  where  $q$  is the electronic charge. Substituting in



Equation 3 and assuming (for generality) the presence of an external static electric field  $\vec{E}_0$  and an internal electric field due to charge separation  $\vec{E}_i$  gives for the holes and electrons, respectively:

$$\frac{d(N_e - N_0)}{dt} = D_e \nabla^2 (N_e - N_0) - \frac{(N_e - N_0)}{\tau} - \mu_e \nabla \cdot [N_e (\vec{E}_i + \vec{E}_0)] + N_g, \quad (4)$$

$$\frac{d(N_h - N_0')}{dt} = D_h \nabla^2 (N_h - N_0') - \frac{(N_h - N_0')}{\tau} + \mu_h \nabla \cdot [N_h (\vec{E}_i + \vec{E}_0)] + N_g, \quad (5)$$

where  $\mu$  is the particle mobility,  $D$  the diffusion coefficient, and  $\tau$  the carrier lifetime (including trapping). The mobility is related to  $\tau$  by  $\mu = q\tau/m^*$ , where  $m^*$  is the effective mass. Coefficient  $D$  can be obtained from the Einstein relation  $D = (kT/q)\mu$ , where  $T$  is the absolute temperature.

In order to simplify the above expressions it is assumed that  $\mu_h \approx \mu_e$  and  $D_h \approx D_e$ , in which case  $N_e \approx N_h$  and  $\vec{E}_i = 0$ . These assumptions can be experimentally verified by the absence of photo-voltaic effects. Assume a steady-state condition

$$\frac{d(N_e - N_0)}{dt} = \frac{d(N_h - N_0')}{dt} = 0.$$

Then Equations 4 and 5 reduce to one equation of the form (for  $\vec{E}_0 = 0$ ),

$$D \nabla^2 (N - N_0) - \frac{(N - N_0)}{\tau} + N_g = 0. \quad (6)$$

In the one-dimensional case under consideration, Equation 6 becomes

$$D \frac{d^2 (N - N_0)}{dz^2} - \frac{(N - N_0)}{\tau} + \frac{\eta P_0 \tau}{h\nu A} \frac{2\alpha}{D\tau} e^{-2\alpha z} = 0. \quad (7)$$

Now let  $\eta P_0 \tau / h\nu A = R$ ,  $1/2\alpha = L_A$  and  $D\tau = L_D^2$ , where  $L_A$  is the attenuation length and  $L_D$  the diffusion length.

The solution of Equation 7 is

$$N - N_0 = \zeta e^{-z/L_D} + R \frac{L_A}{L_D^2 - L_A^2} e^{-z/L_A},$$

where  $\zeta$  is an integration constant obtained by the condition that  $N - N_0$  have no maxima or minima for  $z > 0$ . This is equivalent to saying that the total number of excess particles in the volume ( $At$ )

(where  $t$  is the slab thickness) is the same whether diffusion and recombination, or recombination alone occurs; thus:

$$N - N_0 = \frac{R L_D}{L_D^2 - L_A^2} \left( e^{-z/L_D} - \frac{L_A}{L_D} e^{-z/L_A} \right). \quad (8)$$

With GaAs as an example,  $h\nu = 2.23 \times 10^{-19}$  joules at  $\lambda = 0.9\mu$ ,  $\tau \approx 10^{-9}$  sec, and, assuming  $T = 400^\circ$  to account for heating effects,

$$\begin{aligned} L_A &= 2.12 \times 10^{-7} \text{ m} , \\ L_D &= 5.89 \times 10^{-6} \sqrt{\mu} \text{ m} , \\ R &= 1.13 \times 10^9 P_0 / A \text{ m}^{-2} , \end{aligned} \quad (8a)$$

where  $\mu$  is in  $\text{m}^2/\text{volt sec}$ ,  $P_0$  in watts,  $A$  in  $\text{m}^2$ , and  $\eta = 0.25$  (as a conservative estimate).

If we neglect diffusion; i.e., consider that  $L_D \ll L_A$ , Equation 8 becomes

$$N - N_0 \approx \frac{R}{L_A} e^{-z/L_A}. \quad (9)$$

If diffusion predominates over recombination (high mobility), then  $L_D \gg L_A$  and

$$N - N_0 \approx \frac{R}{L_D} e^{-z/L_D}. \quad (10)$$

Evidently the spatial distribution of electrons (and holes) strongly depends upon the material parameters  $\alpha$ ,  $\mu$ , and  $\tau$ . In any case it is very nearly exponential.

### Modulator Transmission Coefficient

The propagation of a plane electromagnetic wave in a medium of dielectric constant  $\epsilon$ , permeability  $\mu_0$ , and conductivity  $\sigma$  is governed by the wave equation for the electric field  $\vec{E}$ :

$$-\nabla^2 \vec{E} = -j\omega\mu_0(\sigma + j\omega\epsilon)\vec{E}.$$

with assumed sinusoidal time variations of radian frequency  $\omega$  (signal frequency). For waves propagating in the  $z$  direction, the equation can be written

$$\frac{d^2 \vec{E}}{dz^2} + k_1^2 \left( 1 + \frac{\sigma}{j\omega\epsilon} \right) \vec{E} = 0, \quad (11)$$

where  $E$  is transverse to the propagation direction, and  $k_1^2 = \omega^2 \mu_0 \epsilon$ . In a semiconductor containing an inhomogeneous distribution of quasi-free charges in the  $z$  direction, Equation 11 becomes nonlinear, because  $\sigma$  is a function of  $N$  and therefore of  $z$ . In the present case the equation is fortunately amenable to solution in terms of tabulated functions.

To determine the transmission coefficient of the modulator, assume the physical situation shown in Figure 2. A plane electromagnetic wave in region 1 is incident on the slab of material of thickness  $t$ , represented by region 2.

The material contains the distribution of free charge  $N - N_0 = R/L e^{-z/L}$  previously obtained. Free space is assumed on either side of the slab (regions 1 and 3).

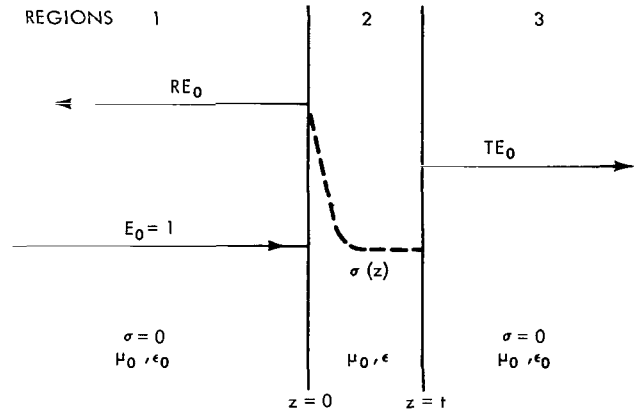


Figure 2—Model for modulator transmission coefficient.

The transmission coefficient  $T$  is determined by solving Equation 11 in each of the three regions and setting appropriate boundary conditions at  $z = 0$  and  $z = t$ . The boundary conditions for normally incident waves are continuity of  $E$  and  $dE/dz$  across the boundary. Appropriate solutions to Equation 11 in regions 1 and 3 are simple traveling waves

$$\begin{aligned} E_1 &= E_0 e^{-jk_0 z} + R e^{jk_0 z}, \\ E_3 &= T e^{-jk_0 z}, \end{aligned} \quad (12)$$

where  $E_0$ ,  $R$ , and  $T$  are constants ( $E_0$  will be set equal to one). A minus sign in the exponential represents a wave propagating in the  $(+z)$  direction. (The time-dependence  $e^{j\omega t}$  is understood.)

In region 2, characterized by the parameters  $\mu_0$ ,  $\epsilon > \epsilon_0$ , and  $\sigma(z)$ , solutions are not so readily obtained, because the material is inhomogeneous. In this region the conductivity  $\sigma$  can be obtained from Drude Theory (Reference 2) as

$$\sigma = \frac{N e^2 / m^*}{\nu_c + j\omega}, \quad (13)$$

where  $N$  is obtained from Equation 9 or 10. The conductivity is generally complex and inhomogeneous, because of the spatial variation of  $N$ . The effective mass  $m^*$  is used here, since the charged particles exist in a crystal lattice.  $\nu_c$  is the effective collision frequency for momentum transfer and is related to the dc mobility  $\mu$  by the relation  $\nu_c = e/m^*\mu$ .

Rationalizing Equation 13 and inserting the result in Equation 11 gives

$$\frac{d^2 E_2}{dz^2} + k_1^2 \left( 1 - j \frac{1}{\omega \epsilon} \left[ \frac{N e^2 / m^*}{\nu_c^2 + \omega^2} (\nu_c - j \omega) \right] \right) E_2 = 0$$

or

$$\frac{d^2 E_2}{dz^2} + k_1^2 \left( \left[ 1 - \frac{\omega_p^2}{\nu_c^2 + \omega^2} \right] - j \frac{\nu_c}{\omega} \frac{\omega_p^2}{\nu_c^2 + \omega^2} \right) E_2 = 0, \quad (14)$$

where  $\omega_p^2 = N e^2 / m^* \epsilon$ , a function of  $z$ . For most semiconductors in the frequency range of interest,  $\nu_c \ll \omega$ . This implies that we can neglect the imaginary term in Equation 14, which represents a small attenuation factor. On the other hand, under certain conditions this term can provide another modulation mechanism, as will be shown later.

Combining Equation 14 with Equation 9 or 10 and applying the above approximations gives

$$\frac{d^2 E_2}{dz^2} + k_1^2 \left( 1 - \frac{R e^2}{L m^* \epsilon \omega^2} e^{-z/L} \right) E_2 = 0 \quad (15)$$

for the propagating electric field. We can put Equation 15 in simpler form by defining the constants  $K^2$  and  $\theta$ , and changing the variable from  $z$  to  $y$ ;

$$K^2 = \frac{R e^2}{L m^* \epsilon \omega^2}, \quad \theta = 2 L k_1, \quad y = \theta K e^{-z/2L}.$$

Then,

$$y^2 \frac{d^2 E_2}{dy^2} + y \frac{dE_2}{dy} - (y^2 - \theta^2) E_2 = 0, \quad (16)$$

which is Bessel's Equation yielding Bessel Functions of imaginary argument and *imaginary order*. The real-valued form of these solutions are the so-called "wedge functions"  $F_\theta(y)$ ,  $G_\theta(y)$  (Reference 3). In the traveling-wave form, the solutions can be written as functions of  $z$ :

$$F_+(z) = [\phi(z) + j \psi(z)] e^{-jk_1 z},$$

$$F_-(z) = [\phi(z) - j \psi(z)] e^{jk_1 z},$$

where

$$\phi(z) = \sum_{m=0}^{\infty} \frac{\theta(-1)^m \left(\frac{\theta K}{2} e^{-z/2L}\right)^m}{m! (m^2 + \theta^2)} I_m(\theta K e^{-z/2L}) ,$$

$$\psi(z) = \sum_{m=1}^{\infty} \frac{m(-1)^m \left(\frac{\theta K}{2} e^{-z/2L}\right)^m}{m! (m^2 + \theta^2)} I_m(\theta K e^{-z/2L}) ,$$

and  $I_m(y)$  is a modified Bessel Function of order  $m$ .

For  $z \gg 2L$ :  $\phi \approx 1/\theta$  and  $\psi \approx 0$ .

If the solutions for all three regions are known, the boundary conditions can be satisfied for the three sets of fields (the incident field is normalized to unity):

$$\begin{aligned} E_1 &= e^{-jk_0 z} + R e^{jk_0 z} , \\ E_2 &= A F_+(z) + B F_-(z) , \\ E_3 &= T e^{-jk_0 z} . \end{aligned} \quad (17)$$

For  $z \gg 2L$ ,  $F_+(z) = 1/\theta e^{-jk_1 z}$  and  $F_-(z) = 1/\theta e^{+jk_1 z}$ .

Hence, at  $z = t \gg 2L$ ,

$$\begin{aligned} \frac{1}{\theta} (A e^{-jk_1 t} + B e^{+jk_1 t}) &= T e^{-jk_0 t} , \\ \frac{-jk_1}{\theta} (A e^{-jk_1 t} - B e^{+jk_1 t}) &= -jk_0 T e^{-jk_0 t} ; \end{aligned} \quad (18)$$

at  $z = 0$ ,

$$\begin{aligned} 1 + R &= A F_+(0) + B F_-(0) , \\ -jk_0 (1 - R) &= A F'_+(0) + B F'_-(0) , \end{aligned} \quad (19)$$

where

$$F'_+(z) = \left[ -jk_1 + \frac{\phi'(z) + j\psi'(z)}{\phi(z) + j\psi(z)} \right] F_+(z) ,$$

$$F_{-}'(z) = \left[ jk_1 + \frac{\phi'(z) - j\psi'(z)}{\phi(z) - j\psi(z)} \right] F_{-}(z) .$$

Solving Equations 18 and 19 for the reflection coefficient R and transmission coefficient T gives

$$R = \frac{2}{n+1} \frac{1 + \frac{n-1}{n+1} e^{-j2k_1 t - j2\beta}}{\left(1 - j \frac{Kn}{n+1} \frac{M'}{M}\right) - \left(1 + j \frac{Kn}{n-1} \frac{M'^*}{M^*}\right) \left(\frac{n-1}{n+1}\right)^2 e^{-j2(k_1 t + \beta)}} - 1 ,$$

$$T = \frac{4n e^{-j\beta}}{(n+1)^2 M_0} \frac{e^{-j2(k_1 - k_0)t}}{\left(1 - j \frac{Kn}{n+1} \frac{M'}{M}\right) - \left(1 + j \frac{Kn}{n-1} \frac{M'^*}{M^*}\right) \left(\frac{n-1}{n+1}\right)^2 e^{-j2(k_1 t + \beta)}} , \quad (20)$$

where

$$M = \phi(\theta K) + j\psi(\theta K) = M_0 e^{j\beta} \begin{pmatrix} \phi(z), & \psi(z) \\ \text{evaluated at } z = 0 \end{pmatrix} ,$$

$$M' = \frac{dM}{d(\theta K)} ,$$

and \* refers to complex conjugate.

In order to give appreciable modulation of T (and/or R),  $\theta K$  should approach a peak value of 3 to 5. This would require  $P_0 \approx 1$  watt at  $0.9\mu$  focused on an area of approximately  $10^{-8} \text{ m}^2$  ( $\approx 0.1\text{-mm}$  beam diameter). Note also that  $\theta K = (2Lk_1/\omega) \sqrt{Re^2/Lm^*} e = 2\sqrt{Re^2\mu_0/m^*}$  is proportional to  $\sqrt{P_0 L/H}$  and independent of the signal frequency. The quantity  $\omega = 4\pi L/\lambda$ , however, is inversely proportional to the signal wavelength. This means that the modulator should operate satisfactorily at any optical signal wavelength. The dependence of the functions  $\phi(y)$  and  $\psi(y)$  on  $\theta$ , however, dictates certain limitations on this statement. The modulation index is independent of optical signal frequency for  $\theta < 0.1$  but decreases monotonically with increasing  $\theta$  up to a value of about 10. Under these conditions the modulator should function equally well at any infrared wavelength (or microwave, for that matter) within the optical transmission limitations of the modulator material. For  $\theta \gg 1$ , large values of  $P_0$  may be needed, in order to obtain significant amplitude modulation. On the other hand, large degrees of phase modulation are usually possible for this range of  $\omega$  values.

## Approximations

Several approximations have been made that may materially affect modulator performance. First, the slab was considered uniformly illuminated by both the pump and signal sources. In a practical situation the pump power must be focused on an area of about  $10^{-8} \text{ m}^2$  (0.1 mm spot),

necessitating the concentration of the signal beam on the same area. Since the slab surface area is probably at least two orders of magnitude larger, transverse diffusion will reduce the electron density over the signal-beam cross-section. This in turn requires more pump power for the same modulation index—as in the uniformly illuminated case. At the same time, the diffusion will setup a transverse distribution of the carrier density, thereby possibly causing nonuniform modulation of the signal beam. The effect may be reduced by the application of a longitudinal static magnetic field in the beam direction; this reduces the transverse diffusion of the free carriers.

Secondly, the imaginary term in Equation 14 has been neglected, as compared with the real part. This is valid as long as  $K < 1$ . When  $K > 1$ , for some value of  $z$ , the imaginary term will predominate (when  $Ke^{-z/L} \approx 1$ ); in which case the solutions are not entirely correct; in this case the spatial region in which this occurs is very thin and the correction to the solutions very slight, especially since the imaginary term is small compared with  $K$ .

In the derivation of Equation 15, the hole density was ignored. If it is included, an additive term appears in  $K^2$  such that,

$$K^2 = \frac{R}{L} \left( \frac{N_e e^2}{m_e^* \epsilon} + \frac{N_h e^2}{m_h^* \epsilon} \right).$$

Since the effective mass of holes,  $m_h^*$ , is usually much larger than that of electrons, the added term may be safely excluded. In any case the holes conveniently tend to increase the value of  $K$  for a given value of  $R$ .

Finally, the assumption of equal diffusion rates for electrons and holes must be verified. The literature does not provide sufficient information to confirm the validity of the assumption; therefore the effect of the assumption on the results will be described here. When the carriers diffuse at different rates, an effective  $p-n$  junction (or at least  $p-p$  or  $n-n$ ) will be set up in the material producing photovoltaic voltages. These, in turn, will alter the individual carrier diffusion constants; at steady state one can define a single ambipolar constant.

With this value (roughly the weighted average) for  $D_h$  and  $D_e$ , the derivation of Equations 9 or 10 would proceed as shown above.

## Operational Limitations

Operational limitations are imposed on the modulator by the need to dissipate optically generated heat and concentrating pump and signal beams on a small enough area. The former problem is resolved by providing a heat sink for the semiconductor that can carry away thermal energy at a rate equal to the pump power plus the small amount of signal power absorbed in the material by the free carriers. The latter problem is solved by proper optical beam collimation and subsequent focusing on the surface of the semiconductor by simple lenses. Superposition of the two beams requires precise adjustment of the above optical components.

## Effect of Applied Fields

The result of applying static magnetic fields has already been described. One interesting aspect of the modulator was its use of applied quasi-static electric fields. The value of the  $L$  can be controlled by applied electric fields parallel to the direction of wave propagation if  $L_D \gg L_A$ . In this case we can write  $L$  in terms of  $L_D$  and  $E_0$  (from Equation 4, including an  $E_0$ ),

$$L = \sqrt{\left(\frac{\mu E_0 \tau}{2}\right)^2 + L_D^2} \pm \left(\frac{\mu E_0 \tau}{2}\right) \begin{matrix} +E_0 > 0 \\ -E_0 < 0 \end{matrix} .$$

This allows another modulation signal to be superimposed upon the primary pump modulation. It also allows close control of the modulation index by variation of the static field  $E_0$ .

## Numerical Example

The theory previously developed is illustrated by the following example.

An available combination of pump source and semiconductor is the GaAs diode laser used as the optical pump in conjunction with the Fe-doped, high-resistivity, GaAs semiconductor developed by RCA. At present, this combination provides pump power levels above 4 watts at a wavelength of  $0.9\mu$ . The semiconductor material has excellent transmission properties to wavelengths of about  $12\mu$ . We choose the  $10.6\mu$   $\text{CO}_2$  laser wavelength as the signal source.

The material parameters, some of which are given in other parts of this paper, are

$$\mu_e = 0.6 \text{ m}^2/\text{v-sec},$$

$$\tau \cong 10^{-9} \text{ sec (values from } 10^{-8} \text{ to } 10^{-10} \text{ have been cited),}$$

$$m^* = 0.047 m_e = 4.28 \times 10^{-32} \text{ kg},$$

$$n = 3.30,$$

$$\eta = 0.25,$$

$$2\alpha = 4.72 \times 10^6/\text{m}.$$

The optical parameters are given by

$$h\nu = 2.21 \times 10^{-19} \text{ joules (for wavelength of } 0.9\mu \text{ microns),}$$

$$k_1 = 2\pi n/\lambda_0 = 1.95 \times 10^6/\text{m}.$$

Using the above values in Equations 8a gives

$$L_A = 2.12 \times 10^{-7} \text{ m},$$

$$L_D = 1.45 \times 10^{-6} \text{ m},$$

and hence

$$L_D \gg L_A .$$



Checking the other approximation gives

$$N_c = 6.24 \times 10^{12} \ll \omega = 1.8 \times 10^{14} .$$

Equation 20 may now be written in terms of these quantities (assuming  $e^{-j2k_1 t} = -1$  or  $t = \pi/4$  for simplicity) as

$$\begin{aligned} R &= 0.465 \frac{(1 - 0.535 e^{-j2\beta})}{\left(1 - j 0.768K \frac{M'}{M}\right) + 0.286 \left(1 + j 1.43 \frac{M'^*}{M^*}\right)} - 1 , \\ T &= \frac{0.126 e^{j0.35\pi}}{\left(1 - j 0.768K \frac{M'}{M}\right) + 0.286 \left(1 + j 1.43 \frac{M'^*}{M^*}\right)} e^{-j2\beta} , \end{aligned} \quad (22)$$

where

$$\theta = \frac{4\pi n}{\lambda} L_D = 5.67$$

and

$$\left. \begin{aligned} \theta K &= 0.7\sqrt{P_0} \\ K &= 0.123\sqrt{P_0} \end{aligned} \right\}, \text{ assuming } A = 10^{-8} \text{ m}^2 .$$

Figure 3 is a graph of computer solutions for Equations 20, also the total phase deviation of the transmitted wave as a function of incident optical pump power. The values shown as a function of  $(\theta K)^2$  are:

1. Absolute value of the transmitted signal  $\vec{E}$  field,
2. Absolute value of the reflected signal  $\vec{E}$  field,
3. Phase shift of the transmitted wave relative to P incident = 0.

Figure 3 shows the general performance of this type of device; from this figure we can find the probable degree and type of modulation, given only the material parameter and incident power variation. For example, the solutions of Equations 22 for the GaAs iron-doped specimen are obtained from Figure 3 by substituting  $\theta K = 0.7P_0$  from Equation 22 for  $y$ . It is evident that 50-degree PM

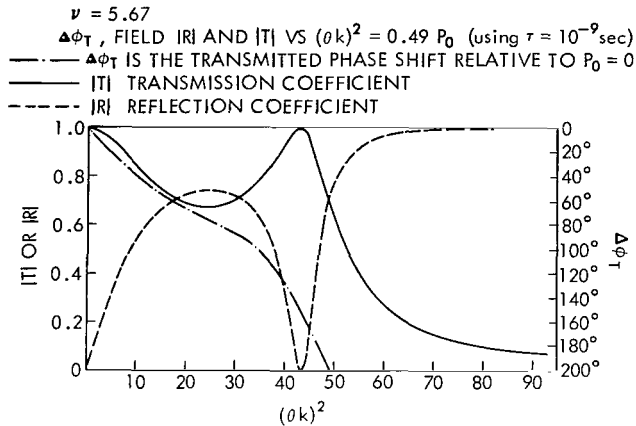


Figure 3—Plotted solutions for reflection and transmission coefficients vs  $y^2$ .

linear modulation can be obtained with a variance in  $P_0$ , at 0.9 microns, of 0 to 25 watts focused on a 0.1-mm diameter spot.

Optical pump power at 0.9 micron at this power level is available in a number of configurations, both with and without refrigeration of the laser diode. The performance of available diodes (Reference 4) indicates optical pump power emitted from single diodes of 200-kw pulse with electron beam pumping, 10w CW at 10°K with efficiencies up to 80 percent, and 10-w pulse at room temperature with efficiencies of 30 percent. These diodes have been switched at frequencies up to 10 GHz. In the

event of insufficient power for a given point of operation, small arrays could easily be switched without distortion or loss of efficiency, since the degree of modulation is independent of the shape or coherence of the incident optical pump wavefront.

## EXPERIMENTAL VERIFICATION

Experiments to verify the preceding theory and computer results are in progress. Indirect verification of this mechanism and its relative merits is already available as a result of experiments (References 5, 6, and 7) by investigators who used free carrier absorption to Q-switch lasers. The resultant data from these experiments closely fit the theory presented here and demonstrate the effect of free-carrier excitation on the reflectivity of a semiconductor. To verify the optically generated free-carrier modulator, it remains only to observe the enhanced effect on longer signal wavelengths with improved drive power of performance.

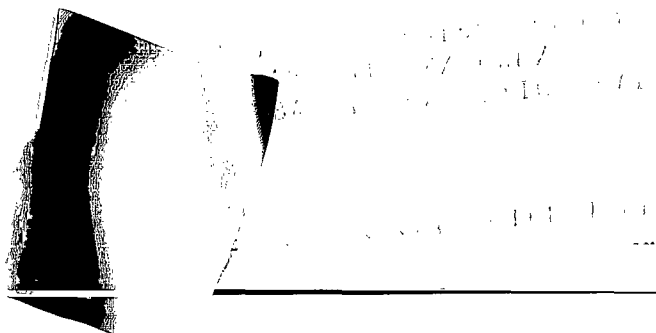
Goddard Space Flight Center  
 National Aeronautics and Space Administration  
 Greenbelt, Maryland, December 5, 1967  
 125-21-02-05-51

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